

- (b) Define prime numbers and composite numbers. Express the definition using symbols. Prove that every integer greater than 1 is either prime or composite. Write first six prime numbers and composite numbers.
- (c) State the Euclidian algorithm. Find the gcd of (330,156) by using Euclidean algorithm.
- (d) Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
- $\forall x \in D$, if x is odd then $x > 0$.
 - $\forall x \in D$, if x is less than 0 then x is even.
 - $\forall x \in D$, if x is even then $x < 0$.
 - $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4,
 - $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.
- (e) Let D be the set of all students at your school, and let $M(s)$ be "s is a math major," let $C(s)$ be "s is a computer science student," and let $E(s)$ be "s is an engineering student." Express each of the following statements using quantifiers, variables, and the predicates $M(s)$, $C(s)$, $E(s)$.
- There is an engineering student who is a math major.
 - Every computer science student is an engineering student.
 - No computer science students are engineering students.
 - Some computer science students are also math majors.
 - Some computer science students are engineering students are some are not.
- (f) Prove that $\sqrt{5}$ is irrational.

3. Attempt the following (any **THREE**) :

[15]

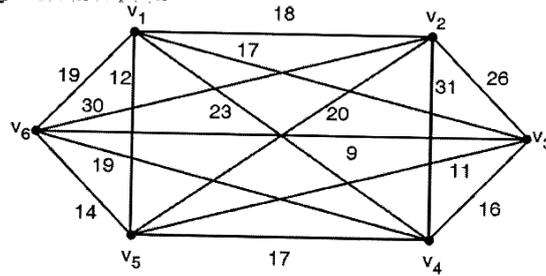
- (a) Define $g: \mathbb{Z} \rightarrow \mathbb{Z}$ by the rule $g(n) = 4n - 5$, for all integers n .
- Is g one-to-one? Prove or give a counterexample.
 - Is g onto? Prove or give a counterexample.
- (b) Prove that $n^3 - 7n + 3$ is divisible by 3, for each integer $n \geq 0$.
- (c) Define : (i) Function (ii) Logarithm (iii) Logarithmic function
(iv) Boolean function (v) Image and Inverse Image
- (d) Define surjective function and inverse function. Find the inverse of the following functions :
- Define $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by the rule $f(n) = 2n$ for all integers n .
 - Define $G: \mathbb{R} \rightarrow \mathbb{R}$ by the rule $G(x) = 4x - 5$ for all real numbers x .
- (e) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be the successor function and let $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be the squaring function. Then $f(n) = n + 1$ for all $n \in \mathbb{Z}$ and $g(n) = n^2$ for all $n \in \mathbb{Z}$.
- Find the compositions $g \circ f$ and $f \circ g$.
 - Is $g \circ f = f \circ g$? Explain
- (f) Find the first four terms of each of the recursively defined sequence
 $S_k = S_{k-1} + 2S_{k-2}$, for all integers $k \geq 2$ $S_0 = 1$, $S_1 = 1$

4. Attempt the following (any **THREE**)

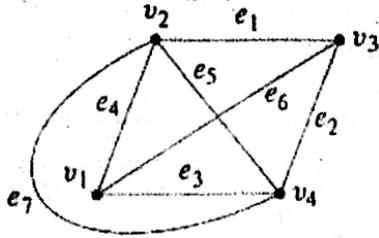
[15]

- (a) Let $S = \{(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)\}$. Find S_+ , the transitive closure of S .
- (b) (i) If R and S are reflexive, is $R \cap S$ reflexive? Why?
(ii) If R and S are symmetric, is $R \cap S$ symmetric? Why?
(iii) If R and S are transitive, is $R \cap S$ transitive? Why?
- (c) Define the following:
- Trail
 - Connected Graph
 - Spanning Tree
 - Hamiltonian Graph
 - Hamiltonian Cycle

- (d) Find Shortest Path from vertex v_1 to all the vertices by applying Dijkstra's Algorithm for the complete weighted graph given below:



- (e) A relation R from \mathbb{R} to \mathbb{R} as follows :
 For all $(x, y) \in \mathbb{R} \times \mathbb{R}$, $x R y \Leftrightarrow y = 2|x|$
 Draw the graphs of R and R^{-1} in the Cartesian plane. Is R^{-1} a function?
 (f) Show that the graph below does not have an Euler circuit.



5. Attempt the following (any THREE)

[15]

- (a) (i) If any seven digits could be used to form a telephone number, how many seven-digit telephone numbers would not have any repeated digits?
 (ii) How many seven-digit telephone numbers would have at least one repeated digit?
 (iii) What is the probability that a randomly chosen seven-digit telephone number would have at least one repeated digit?
- (b) Suppose a person offers to play a game with you. In this game, when you draw a card from a standard 52-card deck, if the card is a face card you win Rs. 3 and if the card is anything else you lose Re. 1. If you agree to play the game, what is your expected gain or loss?
- (c) (i) How many ways can the letters of the word QUICK be arranged in a row?
 (ii) How many ways can the letters of the word QUICK be arranged in a row if the Q and the U must remain next to each other in the order QU?
 (iii) How many ways can the letters of the word QUICK be arranged in a row if the letters QU must remain together but may be in either the order QU or the order UQ?
- (d) A coin is loaded so that the probability of heads is 0.6. Suppose the coin is tossed ten times.
 (i) What is the probability of obtaining eight heads?
 (ii) What is the probability of obtaining at least eight heads?
- (e) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 (i) If five integers are selected from A must at least one pair of the integers have a sum of 9? Explain
 (ii) If four integers are selected from A , must at least one pair of the integers have a sum of 9? Explain.
- (f) How many positive three-digit integers are multiples of 6? What is the probability that a randomly chosen positive three-digit integer is a multiple of 6? What is the probability that a randomly chosen positive three-digit integer is a multiple of 7?

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Paper Discussion Schedule for all Subjects

Date	Day	Timing	Centre
26 Nov. 2018	Monday	9.00 a.m. to 11.00 a.m.	Dadar
26 Nov. 2018	Monday	12.00 p.m. to 2.00 p.m.	Thane
26 Nov. 2018	Monday	6.00 p.m. to 8.00 p.m.	Andheri
27 Nov. 2018	Tuesday	6.00 p.m. to 8.00 p.m.	Borivali