S.Y. B.Sc. (IT) : Sem. III

Applied Mathematics

Prelim Question Paper

Time : 2½ Hrs.] **[Marks : 75**

Instructions : (1) All **questions are** compulsory.
(2) **Make suitable assumptions** wherever **necessary** and **state the assumptions** made.
(3) **Answers** to the **same questions** must be **written together**.
(4) **Numbers** to the **right indicate marks**.
(5) Draw **neat** labeled **diagrams wherever necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt the following (any **THREE**) **[15]**

(a) Verify Caley-Hamilton theorem for the given matrix, also find inverse if exists.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) For different values of k, discuss the following equations:
$x + 2y - z = 0$; $3x + (k + 7)y - 3z = 0$; $2x + 4y + (k - 3)z = 0$

(c) Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(d) Express $\frac{-1}{2} + \frac{\sqrt{3}}{2}$ i in polar form.

(e) Prove that $\left(1 + i\sqrt{3}\right)^8 + \left(1 - i\sqrt{3}\right)^8 = -2^8$

(f) Expand $(1 + \cos x + i \sin x)^n$

2. Attempt the following (any **THREE**) **[15]**

(a) Solve the Differential Equation $dy / dx + x^2y = x^5$

(b) Solve the following Equation $x^2p^2 - 2xpy + (2y^2 - x^2) = 0$

(c) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

(d) Solve $p^2 - py + x = 0$

(e) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = \sin(\log x^2)$

(f) Solve the Differential Equation $(x - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$

3. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^\infty e^{-2t} \sin^2 t\, dt$

(b) Find inverse Laplace Transformation by convolution theorem for

$$f(s) = \frac{s}{(s^2 + 1)\,(s^2 + 4)}$$

(c) Find L[y(t) of the following differential equation:

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-1}$; $y(0) = 1$ and $y'(0) = 2$

(d) Find the inverse Laplace transform of : $\frac{5s + 3}{(s + 1)(s^2 + 2s + 5)}$

(e) Find the Laplace transform of : $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$

(f) Obtain the inverse Laplace transform of each of the given function

$$\frac{(s+1)}{s^3(s-3)^2}$$

4. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$

(b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ by changing polar co–ordinates.

(c) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\,dx\,dy\,dz$

(d) Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy\,dx\,dz$

(e) Evaluate $\iint y\,dx\,dy$ over the area bounded by $y = x^2$, $x + y = 2$

(f) Evaluate $\int_0^3 \int_0^{\sqrt{1+y^2}} \frac{dx\,dy}{(1+x^2+y^2)}$

5. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^{2a} x\left(2ax - x^2\right)^{1/2}\,dx$

(b) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^7 x\,dx$

(c) Show that $\int_0^1 \frac{x^a - x^b}{\log x} = \log\left(\frac{a+1}{b+1}\right)$ using DUIS.

(d) If $y = \int_0^x f(t)\sin\left[a(x-t)\right] \cdot dt$ then show that, $\frac{d^2y}{dx^2} + a^2y = af(x)$

(e) Evaluate : (i) erfc (–x) + erfc (x) (ii) erfc (x) + erfc (x)

(f) Evaluate $\int_0^1 \frac{x^7}{(1-x^4)^{1/2}}\,dx$

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Paper Discussion Schedule for all Subjects

Date	Day	Timing	Centre
21 Oct. 2018	Sunday	8.00 a.m. to 10.00 a.m.	Dadar
21 Oct. 2018	Sunday	1.00 p.m. to 3.00 p.m.	Andheri
21 Oct. 2018	Sunday	3.30 p.m. to 5.30 p.m.	Borivali
21 Oct. 2018	Sunday	1.00 p.m. to 3.00 p.m.	Thane
21 Oct. 2018	Sunday	3.30 p.m. to 5.30 p.m.	Ghatkopar
22 Oct. 2018	Monday	6.00 p.m. to 8.00 p.m.	Nerul