F.Y. B.Sc.(IT) : Sem. II

Numerical and Statistical Methods

Time : 2½ Hrs.] Prelim Question Paper **[Marks : 75**

B.Sc. (IT)

N.B.:
(1) All questions **are** compulsory.
(2) **Make suitable assumptions** wherever **necessary** and **state the assumptions** made.
(3) **Answer** to the **same questions** must be **written together**.
(4) **Numbers** to the **right indicate marks**.
(5) Draw **neat** labeled **diagrams wherever necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt any **THREE** of the following : **[15]**

(a) What is a mathematical model? With the help of a flowchart, explain the of solving an engineering problem.

(b) Explain the terms : (i) Significant figures, (ii) Accuracy, (iii) Precision, (iv) Truncation error, (v) Round-off error

(c) Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (i) the true error and (ii) the true percent relative error for each case.

(d) Use zero- through fourth-order Taylor series expansions to approximate the function

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

from $x_i = 0$ with $h = 1$. That is, predict the function's value at $x_{i+1} = 1$.

(e) Evaluate and interpret the condition number for

$$f(x) = \frac{\sin x}{1+\cos x} \qquad \text{for } x = 1.0001\pi$$

(f) Evaluate e^{-5} using the two formulae :

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots$$

and $$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots}$$

and compare with the true value 6.737947×10^{-3}. Use five terms to evaluate each series and compute true and approximate relative errors as terms are added.

2. Attempt any **THREE** of the following: **[15]**

(a) Find the roots of the equation $x^3 - 12.2x^2 + 7.45x + 42 = 0$ using Regula-Falsi method correct up to 4 decimal places.

(b) Find the roots of the equation

$$x \tan x = 1$$

near 4 using Newton Raphson method correct up to 4 decimal places.

(c) The following table shows the number of students and range of marks. Find the number of students who have secured less than 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	45	32	27	15

(d) Given :

x	1	2	3	4	5	6	7	8
f(x)	0.01	0.004	0.02	0.12	0.15	0.257	0.325	0.231

find f(7.5) using Newton's backward interpolation formula.

(e) Determine the real root of $f(x) = 4x^3 - 6x^2 + 7x - 2.3$ using bisection method correct upto 3 decimal places.

(f) Give log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990 and log 7 = 0.8451. Using Lagrange's formula, find log 47.

3. Attempt any **THREE** of the following: **[15]**

(a) Solve the following simultaneous equations by Gauss -Jordan elimination method:
$2x_1 + 6x_2 - x_3 = -14$
$5x_1 - x_2 + 2x_3 = 29$
$x_3 - 3x_1 - 4x_2 = 4$

(b) Solve the following simultaneous equations by Gauss-Seidel method :
$10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$

(c) Evaluate $\int_0^1 \frac{1-e^{-x}}{x}dx$ using trapezoidal rule and Simpson's 3/8 rule.

(d) Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5+4\cos x}dx$ using Simpson's 3/8th rule.

(e) Solve $\frac{dy}{dx}$ = log (x + y); y(1) = 2 for x = 1.2 and x = 1.4 using Euler's modified method, taking h = 0.2.

(f) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, where y(0) = 1, to find y(0.1) using Runge-Kutta method.

4. Attempt any **THREE** of the following: **[15]**

(a) Fit a straight line to the x and y values in the two rows:

x	1	2	3	4	5	6	7
y	0.5	2.5	2.0	4.0	3.5	6.0	5.2

(b) Fit a second degree parabola for the following :

x	–2.5	–2	–1.5	–0.5	0	0.5	1.5
y	14.32	14.83	15.27	15.47	16.26	16.79	17.23

(c) Use multiple regression to fit the following data :

x_1	x_2	y
0	0	5
2	1	10
2.5	2	9
1	3	0
4	6	3
7	2	27

(d) Maximize 50x + 100y subject to $10x + 5y \le 2500$, $4x + 10y \le 2000$, $x + 1.5y \le 450$ and $x \ge 0$; $y \ge 0$.

(e) A firm makes two types of furniture – chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are processed on three machines M_1, M_2 and M_3 .The time required in hours by each product and total time available in hours per week on each machine are as follows:

MACHINE	CHAIR	TABLE	AVAILABLE TIME
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

How should the manufacturer schedule his production in order maximize contribution ?

(f) An aged person must receive 4000 units of vitamin, 50 units of minerals and 1400 calories a day. A dietician advises to thrive on two foods F1and F2 that cost Rs 4 and Rs 2 respectively per unit of food. It one unit of F1 contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of F2 Contains 100 units of vitamins 2 units of minerals and 40 calories, formulate a linear programming model to minimize the cost of diet.

5. Attempt any **THREE** of the following: **[15]**

(a) The diameter of an electric cable; say X, is assumed to be a continuous random variable with p.d.f. $f(x) = 6x(1 - x),\ 0 \le x \le 1$.

(i) Check that above is p.d.f,

(ii) Determine a number b such that $P(X < b) = P(X > b)$

(b) A random variable x has the following probability distributions :

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k, (ii) Evaluate $P(x < 6)$, $P(x \ge 6)$ and $P(0 < x < 5)$,

(iii) If $P(x \le c) > \frac{1}{2}$, find the minimum value of c, and (iv) Determine the distribution function of x.

(c) The price for a litre of whole milk is uniformly distributed between Rs. 45 and Rs. 55 during July in Mumbai. Give the equation and graph the pdf for X, the price per litre of whole milk during July. Also determine the percent of stores that charge more than Rs. 54 per litre.

(d) What is exponential distribution? Suppose the time till death after infection with Cancer, is exponentially distributed with mean equal to 8 years. If X represents the time till death after infection with Cancer, then find the percentage of people who die within five years after infection with Cancer.

(e) The probability mass function of a random variable X is zero except at the points i = 0, 1, 2. At these points it has the values $p(0) = 3c^3$, $p(1) = 4c - 10c^2$, $p(2) = 5c - 1$ for some $c > 0$.

(i) Determine the value of c.

(ii) Compute the following probabilities, $P(X < 2)$ and $P(1 < X < 2)$.

(iii) Describe the distribution function and draw its graph.

(iv) Find the largest x such that $F(x) < \frac{1}{2}$

(v) Find the smallest x such that $F(x) \ge \frac{1}{3}$

(f) The monthly worldwide average number of airplane crashes of commercial airlines is 2.2. What is the probability that there will be

(i) more than 2 such accidents in the next month?

(ii) more than 4 such accidents in the next 2 months?

Paper Discussion Schedule

Date	Day	Timing	Centre
9 April. 2018	Monday	9 a.m. to 11 a.m.	Dadar
9 April. 2018	Monday	12 p.m. to 2 p.m.	Andheri
9 April.2018	Monday	3 p.m. to 5 p.m.	Borivali
9 April.2018	Monday	5 p.m. to 7 p.m.	Thane
7 April.2018	Saturday	8 a.m. to 10 a.m.	Kalyan
7 April.2018	Saturday	6 p.m. to 8 p.m.	Nerul

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