F.Y. B.Sc.(IT) : Sem. I

Discrete Mathematics

Time : 2½ Hrs.] Prelim Question Paper [Marks : 75

N.B.: (1) All questions are compulsory.
(2) Answer to the same questions must be written together.
(3) Numbers to the right indicate marks.
(4) Draw neat labeled diagrams wherever necessary.

1. Attempt any THREE of the following : [15]

(a) Check whether the following statement for is tautology or contradiction.

$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)$

(b) Use theorems to prove logical equivalence. Supply reason for each step.

$\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

(c) Use truth table to determine whether the argument form is valid. Indicate which columns represents premises and which column represents conclusion and include a sentence explaining how the truth table is supporting your answer.

$p \vee q$
$p \rightarrow \sim q$
$q \rightarrow r$
$\therefore \ r$

(d) Let $W_i = \{x \in R \mid x > i\} = (i, \infty)$ for all non-negative integers i.

(i) $\bigcup_{i=0}^{4} W_i = ?$ (ii) $\bigcap_{i=0}^{4} W_i = ?$

(iii) Are W_0, W_1, W_2, … mutually disjoint? Explain.

(iv) $\bigcup_{i=0}^{n} W_i = ?$ (v) $\bigcap_{i=0}^{n} W_i = ?$

(vi) $\bigcup_{i=0}^{\infty} W_i = ?$ (vi) $\bigcap_{i=0}^{\infty} W_i = ?$

(e) Use an element argument method to prove following statement.
For all sets A and B, $(A \cap B) \cup (A \cap B^c) = A$.

(f) Construct an algebraic proof for following statement.
For all sets A and B, $A \cup (B - A) = A \cup B$.

2. Attempt any THREE of the following : [15]

(a) Write negation for following statement.
(i) If the square of an integer is odd, then the integer is odd.
(ii) $\forall$ real numbers x, if $x^2 \geq 1$ then $x > 0$.

(b) Write negation, converse, inverse and contrapositive of the following statement.
$\forall$ integers a, b and c, if a – b is even and b – c is even, then a – c is even.

(c) Prove that :
If k is any odd integer and m is any even integer, then, $k^2 + m^2$ is odd.

(d) Use the quotient-remainder theorem with d = 3 to prove that the product of any three consecutive integers is divisible by 3.

(e) Prove the following statement by contradiction.
The product of any nonzero rational number any irrational number is irrational.

(f) Rewrite the statement in English without using the symbol $\forall$ or $\exists$ or variables and expressing your answer as simply as possible.
(i) $\forall$ colors C, $\exists$ an animal A such that A is colored C.
(ii) $\exists$ a book b such that $\forall$ people p, p has read b.
(iii) $\forall$ odd integers n, $\exists$ an integer k such that n = 2k + 1.

3. Attempt any THREE of the following : [15]

(a) Let $J_5 = \{0, 1, 2, 3, 4\}$, and define functions $f : J_5 \to J_5$ and $g : J_5 \to J_5$ as follows : For each $x \in J_5$,
$f(x) = (x + 4)^2 \bmod 5$ and $g(x) = (x^2 + 3x + 1) \bmod 5$.
Is f = g? Explain.

(b) Function f is defined on set of real numbers. Is f one to one? Justify your answer.
$f(x) = \dfrac{x}{x^2+1}$, or all real numbers x

(c) Prove the statement by mathematical induction.
$$\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}, \text{ for all integers } n \geq 2.$$

(d) Prove the statement by mathematical induction.
$3^{2n} - 1$ is divisible by 8, for each integer $n \geq 0$.

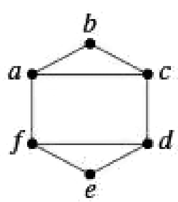
(e) Use iterations to find explicit formula for the following sequence.
$h_k = 2^k - h_{k-1}$, for all integers $k \geq 1$
$h_0 = 1$

(f) Suppose a sequence satisfies the given recurrence relation and initial conditions. find the explicit formula for the sequence.
$r_k = 2r_{k-1} - r_{k-2}$, for all integers $k \geq 2$
$r_0 = 1$, $r_1 = 4$

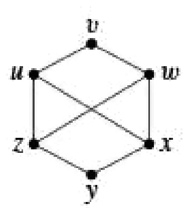
4. Attempt any THREE of the following : [15]

(a) Consider the "divides" relation on each of the following sets A. Draw the Hasse diagram for each relation.
(i) A = {1, 2, 4, 5, 10, 15, 20}
(ii) A = {2, 3, 4, 6, 8, 9, 12, 18}

(b) Determine :
whether G and G′ are isomorphic. If they are, give a function $g : V(G) \to V(G')$ that defines the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

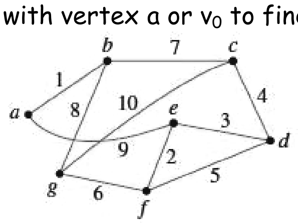


G



G′

(c) Use Prim's algorithm starting with vertex a or v_0 to find a minimum spanning tree.

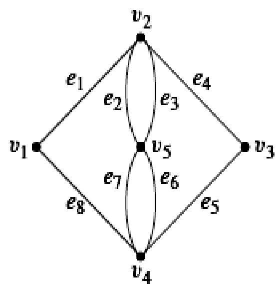


(d) Determine whether the given relation is reflexive, symmetric, transitive, or none of these. Justify your answers.
D is the "divides" relation on Z^+ : For all positive integers m and n, $m\,D\,n \Leftrightarrow m \mid n$.

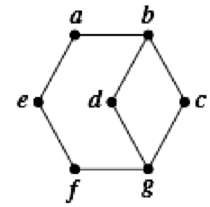
(e) Let A = {–1, 1, 2, 4} and B = {1, 2} and define relations R and S from A to B as follows :
For all $(x, y) \in A \times B$,
$x\,R\,y \Leftrightarrow |x| = |y|$ and
$x\,S\,y \Leftrightarrow x - y$ is even.
State explicitly which ordered pairs are in $A \times B$, R, S, $R \cup S$, $R \cap S$.

(f) Determine whether following graph has Euler circuit and Hamiltonian circuit or not.

(a)



(b)



5. Attempt any THREE of the following : [15]

(a) Explain generalized pigeon hole principle.
A certain college class has 40 students. All the students in the class are known to be from 17 through 34 years of age. You want to make a bet that the class contains at least x students of the same age. How large can you make x and yet be sure to win your bet?

(b) A bakery produces six different kinds of pastry, one of which is eclairs. Assume there are at least 20 pastries of each kind.
(i) How many different selections of twenty pastries are there?
(ii) How many different selections of twenty pastries are there if at least three must be eclairs?
(iii) How many different selections of twenty pastries contain at most two eclairs?

(c) (i) How many distinguishable ways can the letters of the word HULLABALOO be arranged in order?
(ii) How many distinguishable ordering of the letters of HULLABALOO begin with U and end with L?
(iii) How many distinguishable orderings of the letters of HULLABALOO contain the two letters HU next to each other in order?

(d) Use the binomial theorem to expand the expression
$(1 + x)^7$

(e) When a pair of balanced dice are rolled and the sum of the numbers showing face up is computed, the result can be any number from 2 to 12, inclusive. What is the expected value of the sum?

(f) A pool of 10 semifinalists for a job consists of 7 men and 3 women. Because all are considered equally qualified, the names of two of the semifinalist are drawn, one after the other, at random, to become finalists for the job.
What is the probability that both finalists are women?

Paper Discussion Schedule for all Subjects

Date	Day	Timing	Centre
1 Nov. 2016	Tuesday	6.00 p.m. to 7.00 p.m.	Nerul
2 Nov. 2016	Wednesday	9.00 a.m. to 11.00 a.m.	Andheri
2 Nov. 2016	Wednesday	12.00 p.m. to 2.00 p.m.	Dadar
2 Nov. 2016	Wednesday	3.00 p.m. to 5.00 p.m.	Thane

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