S.Y. B.Sc. (IT) : Sem. III

Applied Mathematics

Time : $2\frac{1}{2}$ Hrs.] Prelim Question Paper [Marks : 75

Instructions : (1) All **questions are** compulsory.
(2) **Make suitable assumptions** wherever **necessary** and **state the assumptions** made.
(3) **Answers** to the **same questions** must be **written together**.
(4) **Numbers** to the **right indicate marks**.
(5) Draw **neat** labeled **diagrams wherever necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt the following (any **THREE**) **[15]**

(a) Verify Caley-Hamilton theorem for the given matrix, also find inverse if exists.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) For different values of k, discuss the following equations:
$x + 2y - z = 0;\ 3x + (k + 7)y - 3z = 0;\ 2x + 4y + (k - 3)z = 0$

(c) Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(d) Express $\frac{-1}{2} + \frac{\sqrt{3}}{2}$ i in polar form.

(e) Prove that $\left(1+i\sqrt{3}\right)^8 + \left(1-i\sqrt{3}\right)^8 = -2^8$

(f) Expand $(1 + \cos x + i \sin x)^n$

2. Attempt the following (any **THREE**) **[15]**

(a) Solve the Differential Equation $dy / dx + x^2y = x^5$

(b) Solve the following Equation $x^2p^2 - 2xpy + (2y^2 - x^2) = 0$

(c) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

(d) Solve $p^2 - py + x = 0$

(e) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = \sin(\log x^2)$

(f) Solve the Differential Equation $(x - 4xy - 2y^2)\, dx + (y^2 - 4xy - 2x^2)\, dy = 0$

3. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^\infty e^{-2t} \sin^2 t\, dt$

(b) Find inverse Laplace Transformation by convolution theorem for

$$f(s) = \frac{s}{(s^2+1)\,(s^2+4)}$$

(c) Find L[y(t) of the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-1};\ y(0) = 1 \text{ and } y'(0) = 2$$

(d) Find the inverse Laplace transform of : $\frac{5s+3}{(s+1)(s^2+2s+5)}$

(e) Find the Laplace transform of : $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$

(f) Obtain the inverse Laplace transform of each of the given function

$$\frac{(s+1)}{s^3\,(s-3)^2}$$

4. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)\,(1-y^2)}}$

(b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ by changing polar co–ordinates.

(c) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\,dx\,dy\,dz$

(d) Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy\,dx\,dz$

(e) Evaluate $\iint y\,dx\,dy$ over the area bounded by $y = x^2$, $x + y = 2$

(f) Evaluate $\int_0^3 \int_0^{\sqrt{1+y^2}} \frac{dx\,dy}{(1+x^2+y^2)}$

5. Attempt the following (any **THREE**) **[15]**

(a) Evaluate $\int_0^{2a} x\left(2ax-x^2\right)^{1/2}\,dx$

(b) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^7 x\,dx$

(c) Show that $\int_0^1 \frac{x^a - x^b}{\log x} = \log\left(\frac{a+1}{b+1}\right)$ using DUIS.

(d) If $y = \int_0^x f(t)\sin\left[a(x-t)\right]\cdot dt$ then show that, $\frac{d^2y}{dx^2} + a^2y = af(x)$

(e) Evaluate : (i) erfc (–x) + erfc (x) (ii) erfc (x) + erfc (x)

(f) Evaluate $\int_0^1 \frac{x^7}{(1-x^4)^{1/2}}\,dx$

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